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BOOK REVIEWS

Book Review Editor: Walter Van Assche

Books

A. Bultheel, P. González-Vera, E. Hendriksen, and O. Njåstad, *Orthogonal Rational Functions*, Cambridge Monographs on Applied and Computational Mathematics **5**, Cambridge Univ. Press, Cambridge, UK, 1999, xiv + 407 pp.

The first rational version of Szegő polynomials was given by Djrbashian in the 1960s for the sole purpose of pure mathematical generalization. The real progress came only when the four authors of this book, also known as "the gang of four" among their friends, started to collaborate in the late 1980s. Motivated by problems from different fields ranging from multipoint Padé approximation, moment problems, continued fractions, orthogonal Laurent polynomials, and Schur's algorithm to Nevanlinna–Pick interpolation, the four authors produced a sequence of results that formed a systematic generalization of Szegő polynomials to their rational counterparts. The current book is a summary of some of the most interesting aspects of the latest developments about rational functions orthogonal on the unit circle with prescribed poles.

The book makes extensive use of reproducing kernels and J-unitary and J-contractive matrices in treating recurrence relations. This is why doubled recurrence relations are promoted. These and other algebraic aspects are carefully discussed in the first four chapters. I would have preferred if the restriction "$\alpha_i \in \mathbb{D}$ for $\mathbb{D}$ and $\alpha_i \in \mathbb{U}$ for $\mathbb{U}$" of (1.40) had been repeated in the discussion of (2.1).

Chapter 5 establishes the interpolatory quadrature formulas for integrals on the unit circle. Chapter 6 is about interpolation properties of the orthogonal rational functions and kernels and shows the connection to the Nevanlinna–Pick algorithm. Chapter 7 gives necessary and sufficient conditions for the rational space to be dense in $L_p(\mu)$ or $H_p(\mu)$, which are used in the discussion of the Favard theorem for rational functions in Chapter 8. A partial result of a Favard type theorem for the kernels, which is new even in the polynomial case, is also given. Chapter 9 is the longest among the first 10 chapters. It concerns the asymptotic behavior of the orthogonal rational functions. As long as the speed of convergence of the poles to the unit circle is not too high, polynomial-like results are always possible. These results include the ratio asymptotics, the strong asymptotics, and the weak asymptotics of the rational functions and the kernels. Chapter 10 extends the classical theory of nested circles for moment problems to the rational case. Chapter 11 is devoted to the case when poles are all on the unit circle. Here we face the complexity of possible singularities for integration. Lots of open problems remain in this area. Chapter 12 is a showcase of meaningful applications of the theory of orthogonal rational functions.

The book includes a very extensive introduction to the historical background of the topics, motivations for rational generalization, and a detailed description of every chapter! It also has an interesting conclusion in which the mystery of how the four authors started their successful collaboration is revealed. I find the list of symbols at the beginning of the book very helpful. This is especially important since the book treats orthogonality on the unit circle and on the real line in parallel.

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Overall, this is a very well organized, clearly presented book. It is a fairly comprehensive introduction to the theory of orthogonal rational functions.

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P. K. Kythe, *Computational Conformal Mapping*, Birkhäuser, Boston, 1998, xv + 462 pp.

The mathematics literature is not rich with texts in numerical conformal mapping. The main contributions in this area are (a) the book by von Koppenfels and Stallmann [5], written in 1959; (b) the classic monograph by Gaier [1] which, although written in 1964, is still very relevant; (c) Volume III of Henrici's "*Applied and Computational Complex Analysis*" [2]; (d) the collection of papers on numerical conformal mapping which was edited in 1986 by Trefethen [9]. There are also the two more recent books by Ivanov and Trubetskov [4] and Schinzinger and Laura [7], but these concern mainly applications and do not purport to cover the whole range of numerical conformal mapping techniques. Thus, I was particularly excited to receive this latest addition to the literature of numerical conformal mapping. Unfortunately, my initial anticipation was dampened considerably on closer reading.

The book consists of 15 chapters and 5 appendixes. The topics covered include the Schwarz–Christoffel method, various expansion methods such as the Bergman kernel, the Szegő kernel and the Ritz methods, the mapping of nearly circular regions, the numerical evaluation of Green's function, various integral equation methods such as those of Lichtenstein, Gershgorin, and Warschawski and Stiefel, various numerical methods based on the integral equation formulation of Symm, the integral equation of Kerzman and Stein for the Szegő kernel, the Theodorsen integral equation method, the method of Wegmann, the mapping of doubly and multiply connected regions, airfoils, the treatment of corner and pole-type singularities, and a discussion on the use of conformal mapping for grid generation.

According to the author "*the book is intended to contribute to an effective study program at the graduate level and to serve as a reference book for scientists, engineers and mathematicians in industry*." Unfortunately, I am not convinced that this aim has been fulfilled. In my view, the book is a welcome addition to the existing literature, but only as an auxiliary text that can serve to indicate some of the more recent developments of the subject and to direct those interested to appropriate references. In this context, the extensive bibliography and the introductory chapter (Chap. 0), which sets the historical background of the subject and describes some of the modern developments in the area, are particularly helpful.

On the other hand, I do not think that the book can be used on its own, as a self-contained text, for the detailed study of the subject. My main criticism is that the book is not well organized. As a result, the overall presentation lacks coherence. In particular, the material is often presented in a fragmented manner, by selecting particular sections directly from the original sources, without making an effort to provide the additional explanations and links needed for continuity and ease of understanding. A typical example of this is the material concerning the singularities of the mapping function, in Section 9.5 and Chapter 12. There are also some rather dubious statements. For example, in Section 9.2 and in other parts of the book, the numerical integral equation method of Symm [8], which has nothing to do with orthonormal polynomials, is called the "*orthonormal polynomial method* (*ONP*)" and is, incorrectly, attributed to Rabinowitz [6].

Another criticism concerns the end-of-chapter exercises (problems). These often refer to research items, of rather technical nature, which can hardly be regarded as problems that a student (even an advanced graduate student) would be in a position to tackle. The following examples suffice to illustrate this: (i) Problems 8.10.1, 8.10.2, and 8.10.3 require the proofs of three major theorems and the development of a major algorithm, all taken from research papers. (ii) In Problem 12.7.8 the reader is asked to accomplish an impossible task, i.e., to